The Review on The Charge Distribution on The Conductor Surface

M. Jafari Matehkolaee
Sama technical and vocational training college
Islamic Azad University
Sari Branch, Iran

A. Naderi Asrami
Department of Electrical and Computer Engineering
Babol Noshirvani University of Technology
Babol, Iran

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Abstract
In this paper we have a full review on the surface charge density at disordered conductor surfaces. Basically, reading text books does not resolve ambiguities in this field. As far as is possible, we have tried to the concepts easier to turn. In fact we will answer two questions. One of them is that why do charges tend to go where the curvature is greater? And another, why is surface charge density greater where a surface is more curved?
Keywords: Physics education, charge distribution, conductor.

Introduction

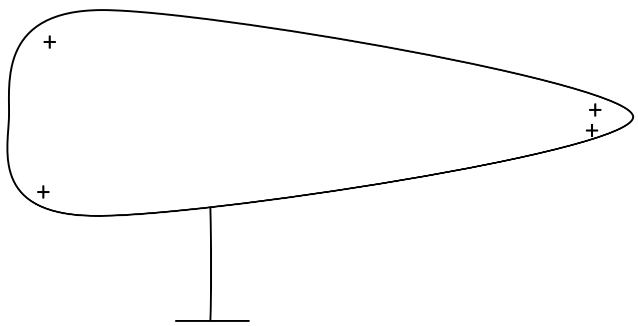
This question has long been on our minds, why is surface charge density greater where a surface is more curved? We have searched for answers to several books and we've been following the precise relationship between surface charge density and curvature. We reached the conclusion that in general there is no unique relationship between conductor curvature and surface charge density. However after reading the paper (McAllister, 1990), we learned that by restricting attention to situation for which the potential is a function of a single variable could be demonstrate that the magnitude of the surface charge density at any point of the conductor surface is proportional to the fourth root of the magnitude of the Gaussian curvature at this location.

Almost for all of the graduate students, it is accepted generally that the electric field strength on of an isolated charged conductor is greatest where the surface curvature is greatest. But it's true that when they started to confront this discussion, there are many problems to understanding it. Because for example the general relationship of the electric field strength and curvature are rather unclear when curvature is finite.

Charge distribution on the surface of the conductor

In this section we are going to answer first question. Why do charges tend to go where the curvature is greater? Indeed, Charges do tend to accumulate at sharp points on conducting bodies these results in a higher surface charge density, which in turn increases the electric field around the point. Surface charges try to spread out as evenly as possible over a conducting surface. Now consider a heuristic example. Suppose we have four similar charges $q_1 = q_2 = q_3 = q_4 = q$ and we are going to distribute them on the following conductor. We wait until the conductor is equipotential by the movement charges. Finally below what we expect:

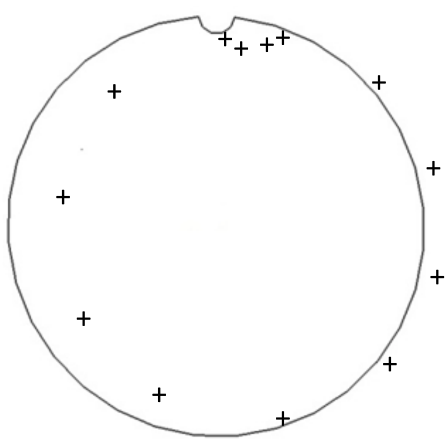
Fig.1



It is now clear that surface charge density at point of A is bigger from B.

Again consider a heuristic example. Think of a sphere with a point sticking out of it. When the charge tries to distribute evenly, more will be deposited on the sharp point than on the rest of the sphere because of the small local increase in surface area of the point. Compare the area of the sphere that is covered by the point with the surface area of the point itself. We think you will see that while the charge is distributed over the surface evenly, compared to the rest of the sphere, there is more charge on the point.

Fig. 2



For a more detailed response we take a different approach. It is interesting that this matter can be pursuing for the insulating body. By means of here by, we consider an insulating sphere with the volume charge density ρ (and total charge Q) and radios R. The value of potential at the center of sphere is $\frac{3}{2}\frac{kQ}{R}$ and on the surface of sphere is $\frac{kQ}{R}$. Then after excite of sphere, charge do tend to spread on the surface. Also consider a solid cube of uniform charge density. What is the ratio of the electrostatic potential at a corner to that at the center? (Boston, 1999)

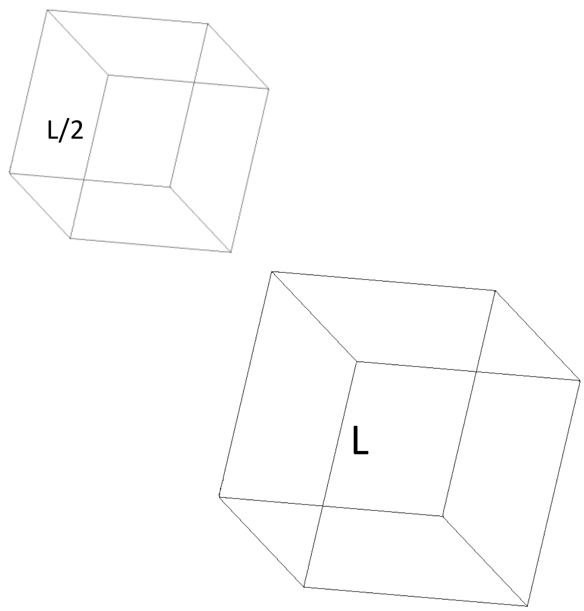


Fig. 3

Suppose the charge density of cube is ρ and also the potential of corner of cube is V_l where l is side of cube and the potential of center of cube is V_l'. By the dimensional analytic we have:

$$V_l \sim \frac{Q}{l} = \rho l^2 \qquad (1)$$

Now we consider a cube with side of $\frac{l}{2}$. Then from (1) we get:

$$V_l = 4V_{\frac{l}{2}} \qquad (2)$$

Note that the volume of first cube is eight times second cube so:

$$V_l' = 8\,V_{\frac{l}{2}} \qquad (3)$$

By the relations (2) and (3) we can conclude:

$$V_l' = 2V_l \qquad (4)$$

Based on the intrinsic property of charges, after stimulation of cube, charge tends to accumulate in the corners. Now we try to respond to the second question. We begin the discussion with a simple example: Now, consider two conducting sphere, a large sphere with radius R and total charge Q, and a small sphere with radius r and total charge q. They are connected by a thin wire and are therefore at the same potential V. The charges of the two spheres are related by the following expression:

$$V = \frac{kq}{r} = \frac{kQ}{R}$$

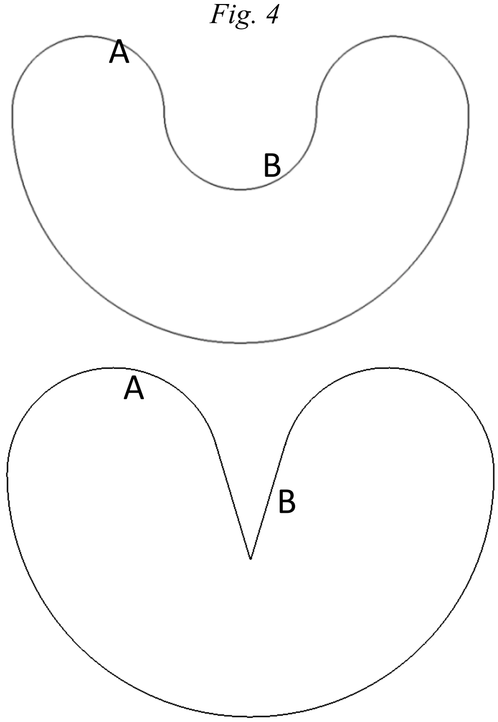
So:

$$\frac{q}{Q} = \frac{r}{R}$$

You can see that the surface charge and therefore the surface charge density and radius are inversely related, so a smaller radius (sharp point in the extreme case) has a higher charge density.

Now we consider an especial example. In the following figure the radius curvature at A is equal to B. we are going to compare charge surface density at A and B.

Fig. 4



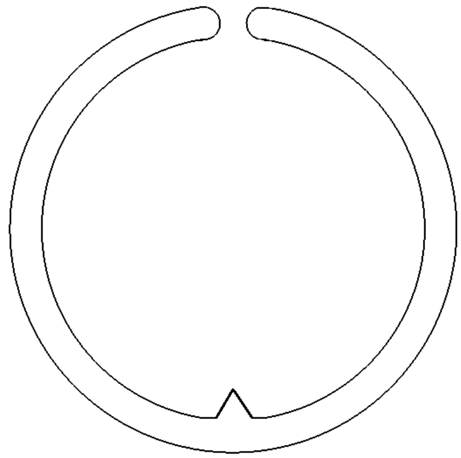


Fig. 5

Here we use from interesting idea in (Price and Crowley, 1985). The electric field inside a closed conducting container is zero; the electric field inside an almost-closed conducting container is almost zero. Similar to the previous study (Price and Cowley, 1985) we consider the almost-closed container that shown in figure (5), so has very weak fields in the hollow that would be in closed by the bottom of this hollow is a small hemispherical pimple. By making the radius of pimple sufficiently small we can guarantee that the maximum surface curvature is on the pimple. But the electric field will not or need not be a maximum on the pimple. Based on the above idea we can conclude that $\sigma_A > \sigma_B$.

Divergence of the electric field on the conductor surface

We investigate divergence on the conductor surface by the Gauss' law. We may use Gauss' law in differential form $\vec{\nabla}.\ \vec{E} = 0$ (charge – free region) assuming the electric field is in the normal direction $\vec{E} = E\hat{n}$ then gives:

$$0 = \vec{\nabla}.\ \vec{E} = \vec{\nabla}.\ (E\hat{n}) = \hat{n}.\vec{\nabla}E + E\vec{\nabla}.\ \hat{n}$$

Using $\hat{n}.\vec{\nabla} = \frac{\partial}{\partial n}$ And rearranging gives:

$$\frac{1}{E}\frac{\partial E}{\partial n} = -\vec{\nabla}.\ \hat{n} \qquad (5)$$

This is a neat expression because a bit of geometry indicates that the divergence of the unit vector field is related to the principal radii of curvature of the associated surface by $\vec{\nabla}.\ \hat{n} = \frac{1}{R_1} + \frac{1}{R_2}$ (Note that, formally, R_1 and R_2 can be positive or negative) substituting this into above then produces:

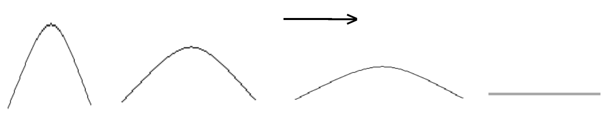
$$\frac{1}{E}\frac{\partial E}{\partial n} = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad (6)$$

This expression is valid in any charge – free region of space, provided we take E and $\hat{n}$ to be the magnitude and direction of the electric field. In this case, R_1 and R_2 represent the principal radii of curvature of the equipotential surface corresponding to the electric field. With a little bit of thought, it is easy to see that the physical interpretation of above equation is that the electric field either gets weaker if the field lines diverge, or stronger if the field lines converge. Now consider two conductors sphere with radii R_1 and R_2 so that $R_1 > R_2$. Can be written $\vec{\nabla}.\ \hat{n} = \frac{2}{R_1}$ and $\vec{\nabla}.\ \hat{n} = \frac{2}{R_2}$ so the electric field on the spheres surface is as following:

$$E = \frac{-\frac{\partial E}{\partial n}}{\vec{\nabla}.\hat{n}} \qquad (7)$$

Clearly, the electric field is stronger on the smaller sphere surface. In general, we show the direction decrease of the divergence and rate of changes of the electric field.

*Fig. 6: The Direction Of Decrease Of The Divergence And Rate Of Changes Of The Ele*ctric field.



Indeed for the flat surface $\frac{\partial E}{\partial n} = 0$ (suppose this surface is infinity and the electric field is uniform) then we have to consider for it $\vec{\nabla}.\ \hat{n} \to 0$ so that E in relation (2) will be definite value. We found an important conclusion, for the pointy charge by the increasing divergence the electric field will be weaker but for the conductor surfaces it is vice versa. If we want extend our discussion for the sharp point on the surface we have $\vec{\nabla}.\ \hat{n} \to \infty$ and $\frac{\partial E}{\partial n} \to \infty$ so $E \to \infty$. This matter for the two-dimensional was investigating by the standard textbooks (Jackson, 1975). But for the sharp point with negative curvature we have $\vec{\nabla}.\ \hat{n} \to -\infty$, $\frac{\partial E}{\partial n} \to$ definite number then $E \to 0$.

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